Graph theory - exercises

1. Find the order, size, minimum degree and maximum degree for graphs: $K_n, P_n, C_n, K_{p,q}$.

2. Show that a finite simple graph with more than one vertex has at least two vertices with the same degree.

3. A graph G has 13 edges and three vertices of degrees 1, 2, and 3, respectively. All remaining vertices have degree 4. How many vertices does G have?

4. The adjacency matrix of a graph G is

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

   Find the degree sequence, order, and size of G. Draw the graph G.

5. Find the incidence matrix and the adjacency matrix of $K_{2,3}$.

6. Does there exist a 3-regular graph of order 7? Does there exist a 3-regular graph of order 8? Explain both answers.

7. Find all nonisomorphic graphs of order four.

8. Find a graph of order four that is isomorphic to its complement. Find a graph of order five that is isomorphic to its complement. Is this possible for graphs of order two or three?